Structural Estimation of Time-varying Spillovers: an Application to International Credit Risk Transmission

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Introduction – Motivation

- Numerous econometric models to gauge financial spillovers (Dungey *et al.*, 2005).
  Main issue: endogeneity of asset prices.

- Among them, **Diebold and Yilmaz** (2009, DY) allow for the construction of time-varying spillovers, from pairwise to systemwise.

- Yet, DY-approach relies on identified SVAR, and empirical research:
  - Either **circumvents** identification-issues (Cholesky, GIRF, Claeys and Vašíček, 2014).
  - Or has attractive identification, but **no time variation** of spillovers **outside of rolling window estimation** (Ando *et al.*, 2018, De Santis and Zimic, 2018, 2019).

Introduction – Overview of our approach

General idea:

- Exploit time-varying FEVDs as measures of spillovers (Diebold-Yilmaz).

Data:

- 9 EZ sovereign CDS: BE DE ES FR GR IE IT NL PT.
- 7 corresponding EZ bank CDS, apart for GR and IE. Daily since 2008.

Econometric roadmap:

1. Filter CDS series from **common shocks**.
2. Estimate a SVAR-GARCH on the residuals, **identified by heteroskedasticity**.
3. **Economic identification** by contribution, validated by matching structural shocks with historical events.
4. TV variances from GARCH processes allow construction of TV FEVDs.

Introduction – Preview of main results

1. **Methodological Contribution**
   - Identified orthogonal shocks even in a **16-variable** SVAR.
   - Our contagion indices appear **more reactive** than other methods used in the literature (Granger-causality).
   - Pairwise spillovers **better identify** the sources of shocks.
2. **Economic Validation**
   - Our estimates match both **the narratives of spillovers** during the EZ debt crisis...
   - ...and **the theoretical channels** of credit risk spillovers.

Model – Identification by Heteroskedasticity

$$\begin{array}{ccc} \text{Reduced form VAR} & & \text{Structural VAR} \\ \mathbf{y}_t = \mathbf{A(L)}\ \mathbf{y}_{t-1} + \boldsymbol{\mu}_t & \rightarrow & \boldsymbol{B}_0 \mathbf{y}_t = \mathbf{B(L)}\ \mathbf{y}_{t-1} + \boldsymbol{\epsilon}_t \end{array}$$

Assume:

$$E(\boldsymbol{\mu}_t \boldsymbol{\mu}_t') = \begin{cases} \boldsymbol{\Sigma}_1 & \text{if t} \leq \text{T} \\ \boldsymbol{\Sigma}_2 & \text{if t} > \text{T} \end{cases} \quad E(\boldsymbol{\epsilon}_t \boldsymbol{\epsilon}_t') = \begin{cases} \boldsymbol{I} & \text{if t} \leq \text{T} \\ \boldsymbol{\lambda} & \text{if t} > \text{T} \end{cases}$$

Then, fully identified system:

$$\boldsymbol{\Sigma}_1 = \boldsymbol{B}_0 \boldsymbol{B}_0^{-1} \text{ and } \boldsymbol{\Sigma}_2 = \boldsymbol{B}_0 \boldsymbol{\lambda} \boldsymbol{B}_0^{-1}$$

Works also with more than two volatility regimes (as in our case)

Model – SVAR-GARCH

Assume:

$$\epsilon_{k,t} = \sigma_{k,t|t-1} e_{k,t} \quad \text{where} \quad \boldsymbol{e}_t \sim \text{i.i.d. N}(\boldsymbol{0}, \boldsymbol{I}_N) \quad \text{and} \tag{1}$$

$$\sigma^2_{k,t|t-1} = (1 - \gamma_k - g_k) + \gamma_k (\epsilon_{k,t-1})^2 + g_k \sigma^2_{k,t-1|t-2} \tag{2}$$

Model – SVAR-GARCH

Assume:

$$\epsilon_{k,t} = \sigma_{k,t|t-1} e_{k,t} \quad \text{where} \quad \boldsymbol{e}_t \sim \text{i.i.d. N}(\boldsymbol{0}, \boldsymbol{I}_N) \quad \text{and} \tag{1}$$

$$\sigma^2_{k,t|t-1} = (1 - \gamma_k - g_k) + \gamma_k (\epsilon_{k,t-1})^2 + g_k \sigma^2_{k,t-1|t-2} \tag{2}$$

Then, we can express the reduced form shocks as:

$$\boldsymbol{\mu}_t = \boldsymbol{B}_0^{-1} \boldsymbol{\lambda}^{\frac{1}{2}}_{t|t-1} \boldsymbol{e}_t \tag{3}$$

where:

$$\boldsymbol{\lambda}_{t|t-1} = \begin{bmatrix} \sigma^2_{1,t|t-1} & & 0 \\ & ... & \\ 0 & & \sigma^2_{N,t|t-1} \end{bmatrix} \tag{4}$$

Model – SVAR-GARCH

Assume:

$$\epsilon_{k,t} = \sigma_{k,t|t-1} e_{k,t} \quad \text{where} \quad \boldsymbol{e}_t \sim \text{i.i.d. N}(\boldsymbol{0}, \boldsymbol{I}_N) \quad \text{and} \tag{1}$$

$$\sigma^2_{k,t|t-1} = (1 - \gamma_k - g_k) + \gamma_k (\epsilon_{k,t-1})^2 + g_k \sigma^2_{k,t-1|t-2} \tag{2}$$

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where:

$$\boldsymbol{\lambda}_{t|t-1} = \begin{bmatrix} \sigma^2_{1,t|t-1} & & 0 \\ & ... & \\ 0 & & \sigma^2_{N,t|t-1} \end{bmatrix} \tag{4}$$

Statistical identification:

$$\boldsymbol{\Sigma}_{u,t|t-1} = \boldsymbol{B}_0^{-1} \boldsymbol{\lambda}_{t|t-1} \boldsymbol{B}_0^{-1\prime} \tag{5}$$

Model – Diebold-Yilmaz Index (sum-up)

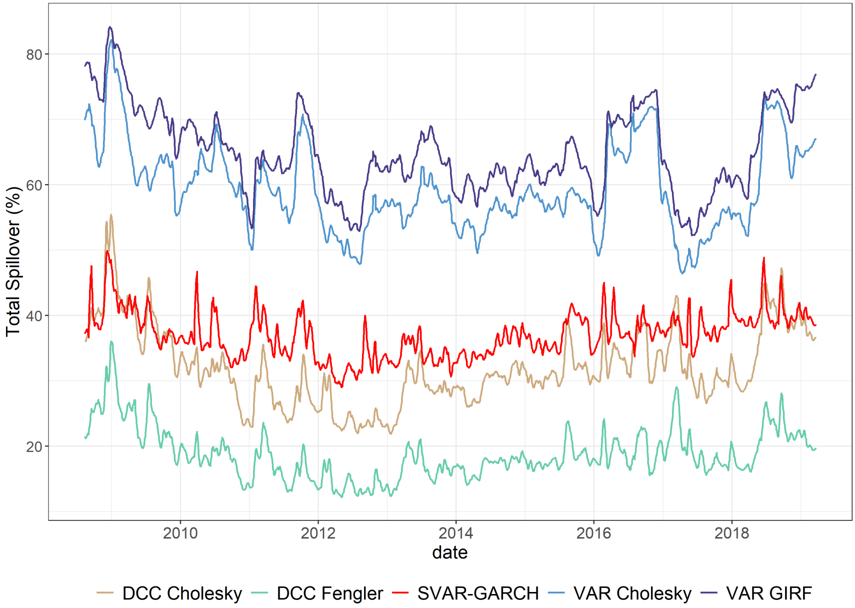
1. VAR: $\mathbf{y}_t = \mathbf{A(L)}\ \mathbf{y}_{t-1} + \boldsymbol{\mu}_t$
2. Identification: $\boldsymbol{B}_0\mathbf{y}_t = \mathbf{B(L)}\ \mathbf{y}_{t-1} + \boldsymbol{\epsilon}_t$ and $E_{t-1}(\boldsymbol{\epsilon}_t\boldsymbol{\epsilon}_t') = \boldsymbol{\lambda}_{t|t-1}$
3. Use of FEVD as connectedness tables:

	y_1	y_2	$\cdots$	y_N	To Others
y_1	d_{11}^H	d_{12}^H	$\cdots$	d_{1N}^H	$\sum_{j=1}^N d_{1j}^H,\ j \neq 1$
y_2	d_{21}^H	d_{22}^H	$\cdots$	d_{2N}^H	$\sum_{j=1}^N d_{2j}^H,\ j \neq 2$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
y_N	d_{N1}^H	d_{N2}^H	$\cdots$	d_{NN}^H	$\sum_{j=1}^N d_{Nj}^H,\ j \neq N$
From Others	$\sum_{i=1}^N d_{i1}^H$ $i \neq 1$	$\sum_{i=2}^N d_{i2}^H$ $i \neq 2$	$\cdots$	$\sum_{i=3}^N d_{i3}^H$ $i \neq N$	$\frac{1}{N}\sum_{i,j=1}^N d_{ij}^H$ $i \neq j$

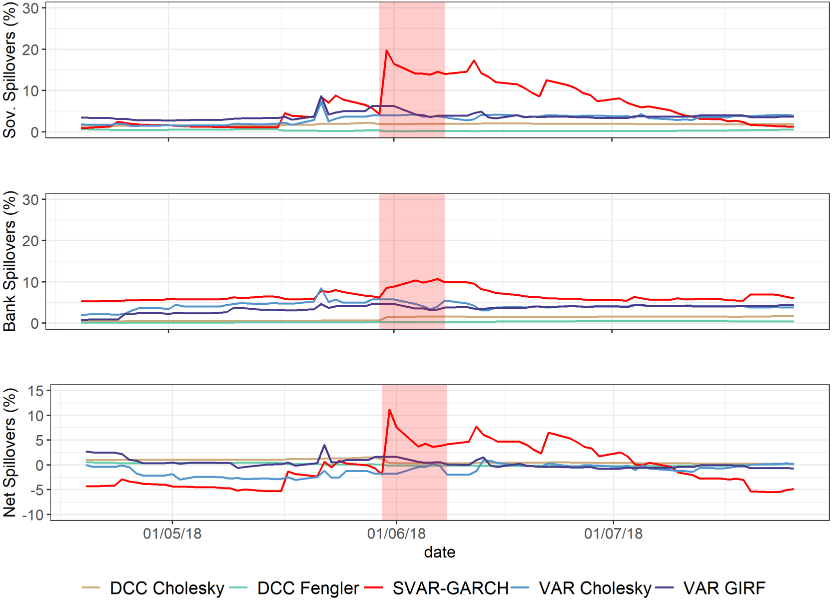
Economic Identification – FEVD

	BE_bk	FR_bk	DE_bk	IT_bk	NL_bk	ES_bk	PT_bk	DE	BE	FR	GR	NL	ES	IT	PT	IE
BE_bk	81%	1%	3%	0%	2%	0%	0%	2%	1%	1%	0%	1%	0%	0%	0%	1%
FR_bk	0%	54%	5%	2%	0%	6%	0%	2%	16%	11%	1%	2%	4%	6%	3%	5%
DE_bk	3%	9%	78%	3%	2%	3%	1%	4%	11%	7%	2%	4%	11%	8%	10%	10%
IT_bk	3%	5%	1%	83%	17%	16%	4%	0%	1%	0%	5%	0%	2%	2%	3%	0%
NL_bk	1%	2%	0%	5%	61%	1%	1%	3%	3%	3%	3%	7%	19%	4%	9%	4%
ES_bk	2%	23%	3%	1%	1%	56%	1%	10%	4%	3%	1%	1%	1%	1%	1%	4%
PT_bk	2%	1%	0%	1%	0%	1%	84%	0%	0%	1%	2%	0%	1%	1%	2%	1%
DE	4%	1%	1%	0%	2%	2%	0%	70%	13%	8%	2%	10%	4%	5%	3%	6%
BE	0%	2%	0%	0%	1%	1%	1%	1%	34%	0%	1%	3%	1%	2%	2%	5%
FR	1%	2%	0%	0%	1%	2%	1%	0%	5%	61%	1%	1%	1%	3%	1%	1%
GR	0%	0%	0%	0%	1%	0%	0%	1%	0%	0%	76%	0%	1%	0%	0%	1%
NL	1%	0%	0%	1%	1%	2%	0%	2%	9%	3%	1%	68%	1%	2%	1%	7%
ES	1%	0%	6%	2%	8%	8%	1%	1%	0%	1%	0%	1%	50%	7%	15%	1%
IT	0%	0%	0%	0%	1%	1%	0%	1%	1%	1%	2%	1%	2%	58%	3%	2%
PT	0%	0%	0%	0%	0%	0%	1%	0%	1%	0%	2%	1%	2%	1%	40%	0%
IE	1%	0%	0%	1%	0%	0%	3%	1%	0%	1%	0%	0%	1%	1%	8%	53%

Model Comparison – Total Spillovers



Model Comparison – Identification (i)



Model Comparison – Identification (ii)

	DCC Fengler	DCC Cholesky	VAR GIRF	VAR Cholesky	**SVAR - GARCH**
(i) Subset of sovereign events	11.0	22.0	44.0	39.0	**78.0**
(ii) Total sovereign events	30.8	33.3	33.3	38.5	**64.1**
(iii) Total sovereign and bank events	34.2	36.8	39.5	47.4	**68.4**
(iv) Candelon et al. (2011)	36.3	45.4	36.4	44.6	**63.6**
(v) Alexandre et al. (2016)	12.5	25.0	75.0	0.5	**75.0**

Note: This table reports the percentage of correct event identifications by each model.

Economic Validation – Theoretical channels (i)

- Do estimates also match with **theoretical contagion channels**?
- We turn to panel analysis, regressing:
  - $\bar{\omega}_{i \to j,t}$ quarterly average pairwise spillover from i to j.
  - On $\boldsymbol{D}_{ijt}$ distance variables between i and j, e.g.: similarity in debt/GDP.
  - And on $\boldsymbol{E}_{j \to i,t}$, exposure variables of j on i, e.g.: trade links.
- We assess **various potential channels**, such as:
  - Trade links between sovereigns.
  - Similar portfolio channel between banks.
  - Implicit bailout channel from banks to sovereigns.
  - Balance sheet channel from sovereigns to banks...

Economic Validation – Theoretical channels (ii)

$$\bar{\omega}_{i\to j,t} = \beta_i + \boldsymbol{D}_{ijt}\boldsymbol{\beta}_1 + \boldsymbol{E}_{j\to i,t}\boldsymbol{\beta}_2 + \alpha_t + \epsilon_{ij,t} \quad (6)$$

Regression		**Expected channels**	**Significant channels**
Sov. - Sov.		Distance: Business Cycle, D/GDP Exposure: Trade, Investment	D/GDP (+), Trade (+), Investment (+)
Bank - Bank		Distance: NPLs, Capital ratios Exposure: Sim. portfolios, bank claims	Capital ratios (+), Sim. portfolios (+)
Bank - Sov.	*High debt*	Vulnerability: Capital ratio, D/GDP, Current account, GDP growth Exposure: domestic sovereign / domestic NFC	Cap ratio (-), D/GDP (+)
	Low debt		Cap ratio (-), D/GDP (-), NFC (+)
Sov. - Bank	*High debt*	Vulnerability: NPLs, Capital ratio, Liquidity Exposure: domestic sovereign / domestic NFC	NPLs (+), Liquidity (-), Sovereign debt (+)
	Low debt		Liquidity (-), Sovereign debt (+), NFCs (+)

Conclusion

- Model: **Structural version** of DY based on a SVAR-GARCH with **up-to-date spillovers**.
- Methodological Results:
  - **Economic identification** enabled, even in a 16-variable system.
  - **Outperforms other models** in terms of timeliness and narrative fit.
  - Validation further supported by regression analysis: **estimates match theoretical channels**.
- Further research:
  - **Trade-off** between daily reactive estimates and time-varying $\boldsymbol{B}_0$.
  - Fewer constraints than usual SVAR: could be used for alternative datasets (e.g. liquidity contagion).

THANK YOU

BACKUP SLIDES

Annex – FEVD build-up

- With the MA representation of $\boldsymbol{Y}_t$:

$$\boldsymbol{Y}_{t+H} - \boldsymbol{Y}_{t+H|t} = \sum_{i=0}^{H-1} \boldsymbol{\Theta}_i \boldsymbol{\epsilon}_{t+H-i} \tag{7}$$

- FEVD is a function of MSPE:

$$\begin{aligned} \boldsymbol{MSPE}_t(H) &= E_t(\boldsymbol{Y}_{t+H} - \boldsymbol{Y}_{t+H|t})(\boldsymbol{Y}_{t+H} - \boldsymbol{y}_{t+H|t})' \\ &= \sum_{i=0}^{H-1} \boldsymbol{\Theta}_i \boldsymbol{\lambda}^*_{t+H-i|t} \boldsymbol{\Theta}'_i \end{aligned} \tag{8}$$

- So connectedness may come from 2 sources, propagation mechanisms ($\boldsymbol{\Theta}_i = \boldsymbol{J}\boldsymbol{A}^i\boldsymbol{J}'\boldsymbol{B}_0^{-1}$) or volatility ($\boldsymbol{\lambda}^*_{t+H-i|t}$). Volatility being projected at each t with the GARCH structure of the SVAR.

Annex – Theoretical channels, between sovereigns

$$\bar{\omega}_{i\to j,t}(h) = \beta_i + \alpha_t + \beta_2 d_{ij,t}^{GDP} + \beta_3 d_{ij,t}^{\frac{D}{GDP}} + \beta_4 \text{exposure}_{j\to i,t}^{k} + \epsilon_{ij,t} \quad (9)$$

	(1)	(2)	(3)
Similar BC	−0.00005	−0.002	−0.001
	(0.001)	(0.001)	(0.001)
Similar D/GDP	0.021***	0.007***	0.014***
	(0.002)	(0.002)	(0.002)
Trade exposure		0.433***	
		(0.020)	
Investment exposure			0.236***
			(0.028)
Time fixed effects?	*Yes*	*Yes*	*Yes*
i fixed effects?	*Yes*	*Yes*	*Yes*
Observations	3,240	3,240	3,171
R^2	0.448	0.598	0.490
Adjusted R^2	0.438	0.591	0.481

*p<0.05; **p<0.01; ***p<0.001

Annex – Theoretical channels, between banks

$$\bar{\omega}_{i\to j,t}(h) = \beta_i + \alpha_t + \beta_2 d_{ij,t}^{NPL} + \beta_3 d_{ij,t}^{Lev.R.} + \beta_4 \text{exposure}_{j\to i,t}^{k} + \epsilon_{ij,t} \qquad (10)$$

	(1)	(2)	(3)
Similar NPLs	−0.001	−0.003	−0.001
	(0.003)	(0.003)	(0.003)
Sim. Capital ratios	0.037***	0.029**	0.040***
	(0.009)	(0.010)	(0.010)
Similar portfolio		7.304***	
		(1.355)	
Bank claims			−0.013
			(0.009)
Time fixed effects?	*Yes*	*Yes*	*Yes*
i fixed effects?	*Yes*	*Yes*	*Yes*
Observations	1,812	1,812	1,812
R^2	0.434	0.439	0.435
Adjusted R^2	0.417	0.422	0.417

*p<0.05; **p<0.01; ***p<0.001

Annex – Theoretical channels, bank to sovereign

$$\bar{\omega}_{bank_i \to sov_i,t}(h) = \beta_0 + \alpha_t + \beta_1 v^{Lev.R.}_{bank_i,t} + \beta_2 v^{D/GDP}_{sov_i,t} + \beta_3 v^{CA}_{sov_i,t} + \beta_4 v^{g_{GDP}}_{sov_i,t} + \beta_5 \text{exposure}^{k}_{sov_i \to bank_i,t} + \epsilon_{bank_i,sov_i,t} \quad (11)$$

	High debt countries			*Low debt countries*		
	(1)	(2)	(3)	(4)	(5)	(6)
Capital	−0.91***	−0.91***	−1.01**	−5.01***	−5.07***	−3.88***
	(0.21)	(0.21)	(0.31)	(1.04)	(1.05)	(1.07)
Debt to GDP	1.00***	1.01***	0.88***	3.74**	3.56**	3.50**
	(0.17)	(0.17)	(0.21)	(1.29)	(1.28)	(1.22)
Current Account	0.13	0.13	0.23	−0.63	−0.62	0.16
	(0.20)	(0.26)	(0.28)	(1.26)	(1.28)	(1.31)
GDP growth	−0.06	−0.05	−0.05	−0.29	−0.31	−0.28
	(0.22)	(0.23)	(0.22)	(0.66)	(0.66)	(0.68)
Sov. exposure		0.06			2.94	
		(1.92)			(9.63)	
Non-bank exposure			−0.88			20.52***
			(1.50)			(6.02)
Time fixed effects?	*Yes*	*Yes*	*Yes*	*Yes*	*Yes*	*Yes*
Observations	174	174	174	123	123	123
R^2	0.74	0.74	0.74	0.93	0.93	0.94
Adjusted R^2	0.64	0.64	0.64	0.89	0.89	0.90

*p<0.05; **p<0.01; ***p<0.001

Annex – Theoretical channels, sovereign to bank

$$\bar{\omega}_{sov_i \to bank_i,t}(h) = \beta_0 + \alpha_t + \beta_1 v^{NPL}_{bank_i,t} + \beta_2 v^{Lev.R.}_{bank_i,t} + \beta_3 v^{Liq.R.}_{bank_i,t} + + \beta_5 \text{exposure}^k_{bank_i \to sov_i,t} + \epsilon_{sov_i,bank_i,t} \quad (12)$$

	High debt countries			*Low debt countries*		
	(1)	(2)	(3)	(4)	(5)	(6)
NPLs	2.61**	2.24**	2.74**	−0.10	−0.12	−0.19
	(0.81)	(0.81)	(0.84)	(0.18)	(0.17)	(0.17)
Capital	−1.38	−0.80	−2.49	−0.06	0.34	0.41
	(0.77)	(0.76)	(1.31)	(0.23)	(0.23)	(0.31)
Liquid assets	−4.03***	−6.17***	−4.60***	−0.92***	−0.11	−0.43
	(0.48)	(1.01)	(0.69)	(0.18)	(0.29)	(0.29)
Exposure domestic gov. debt		0.18**			0.18***	
		(0.07)			(0.05)	
Exposure domestic NFCs			−0.04			0.05**
			(0.03)			(0.02)
Time fixed effects?	*Yes*	*Yes*	*Yes*	*Yes*	*Yes*	*Yes*
Observations	174	174	174	121	121	121
R^2	0.54	0.55	0.54	0.88	0.89	0.88
Adjusted R^2	0.37	0.38	0.37	0.80	0.82	0.81

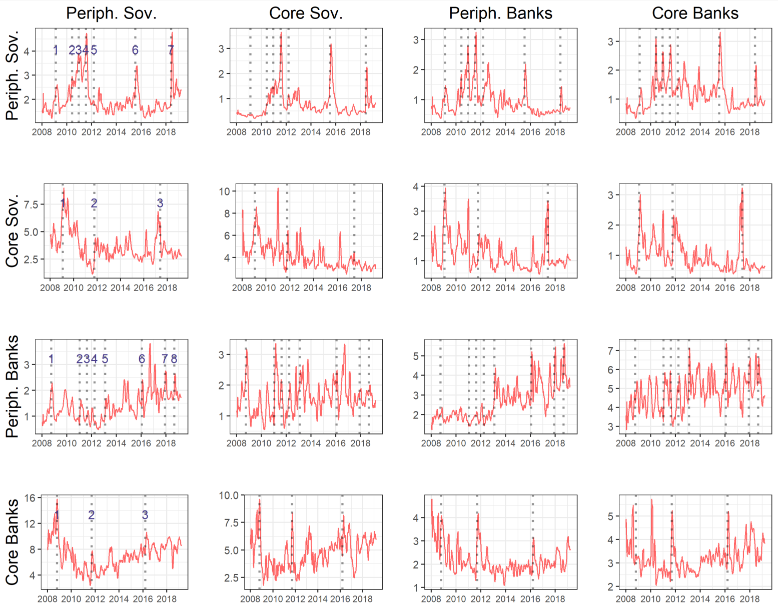
*p<0.05; **p<0.01; ***p<0.001

Annex – Granger causality

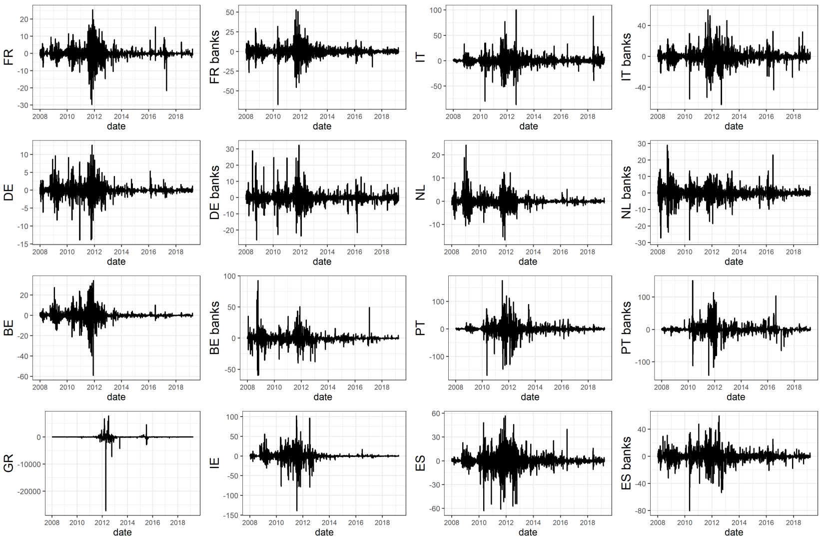
H_0 : **SVAR-GARCH does not Granger cause:**	**F - test**	**p-value**
Rolling window estimated models		
VAR Cholesky	14.91	3.627e-07***
VAR GIRF	6.0492	0.002391 **
GARCH-related models		
DCC Cholesky	1.0527	0.3491
DCC Fengler	1.3641	0.2558

H_0 : **SVAR-GARCH is not Granger caused by:**	**F - test**	**p-value**
Rolling window estimated models		
VAR Cholesky	0.5159	0.597
VAR GIRF	0.9483	0.3875
GARCH-related models		
DCC Cholesky	0.4206	0.6567
DCC Fengler	8.8071	0.0001539***

Annex – Pairwise spillovers



Annex – Inputs

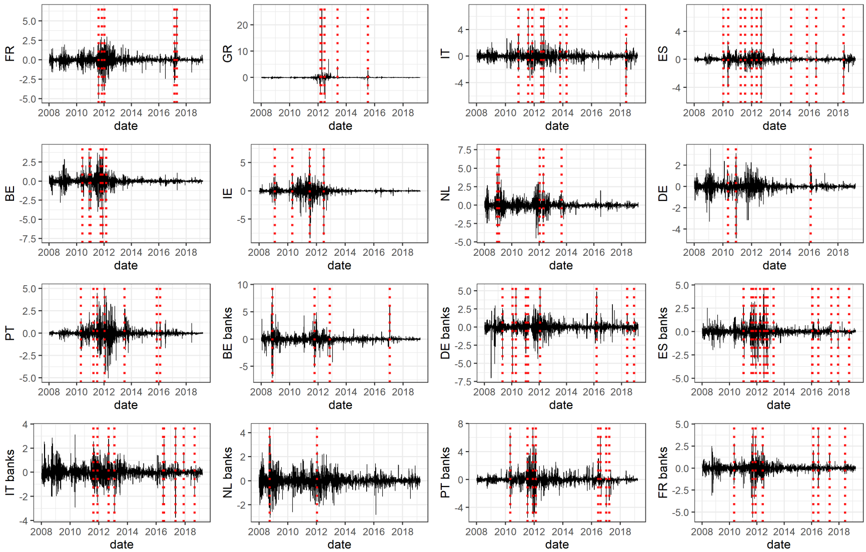


Annex – Granger causality

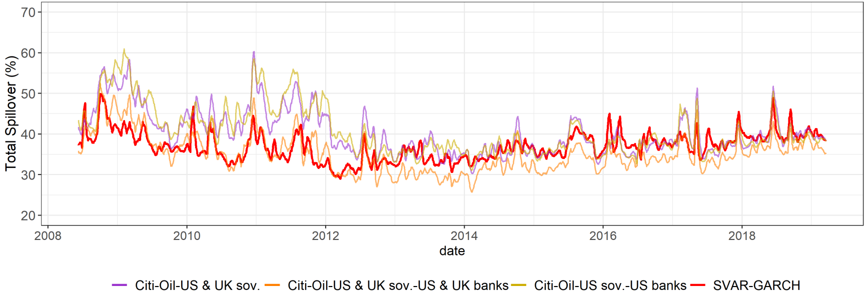
H_0 : **SVAR-GARCH does not Granger cause:**	**F - test**	**p-value**
Rolling window estimated models		
VAR Cholesky	14.91	3.627e-07***
VAR GIRF	6.0492	0.002391 **
GARCH-related models		
DCC Cholesky	1.0527	0.3491
DCC Fengler	1.3641	0.2558

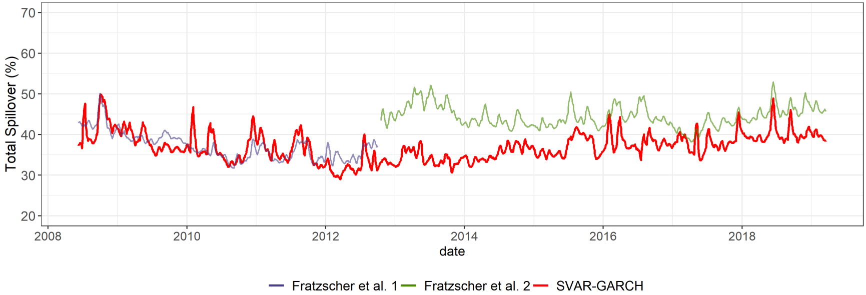
H_0 : **SVAR-GARCH is not Granger caused by:**	**F - test**	**p-value**
Rolling window estimated models		
VAR Cholesky	0.5159	0.597
VAR GIRF	0.9483	0.3875
GARCH-related models		
DCC Cholesky	0.4206	0.6567
DCC Fengler	8.8071	0.0001539***

Annex – Events



Annex – Robustness





Annex – Test identification

h under H_0	$Q_1(1)$	df	p-value
1	124.3405	1	$< 10^{-5}$
2	113.4685	1	$< 10^{-5}$
3	85.0733	1	$< 10^{-5}$
4	66.6269	1	$< 10^{-5}$
5	60.7231	1	$< 10^{-5}$
6	46.2298	1	$< 10^{-5}$
7	38.0658	1	$< 10^{-5}$
8	35.8007	1	$< 10^{-5}$
9	25.3033	1	$< 10^{-5}$
10	16.2284	1	5.615e-05
11	13.3168	1	0.000263
12	12.6034	1	0.000385
13	517.7083	1	$< 10^{-5}$
14	185.0355	1	$< 10^{-5}$
15	154.8558	1	$< 10^{-5}$